Infinite trees of matroids

Nathan Bowler[1]

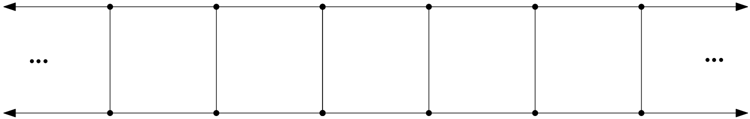
January 2013

[1] Joint work with Johannes Carmesin

Examples of infinite matroids

Two matroids from a (locally finite) graph G, given in terms of their circuits:

Finite cycle matroid	$M_{FC}(G)$:	finite cycles
Topological cycle matroid	$M_C(G)$:	topological circles in $\lvert G \rvert$



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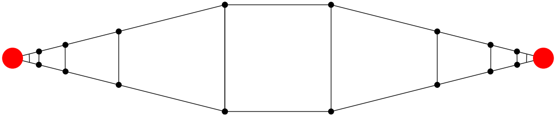

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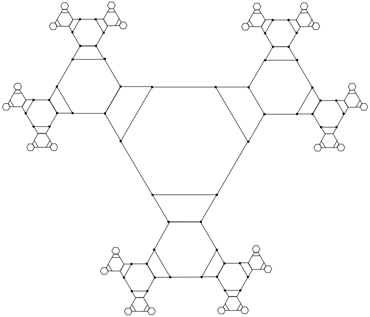

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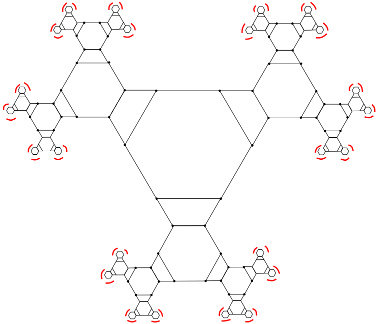

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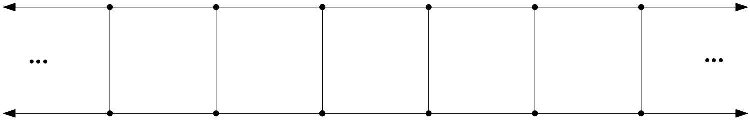

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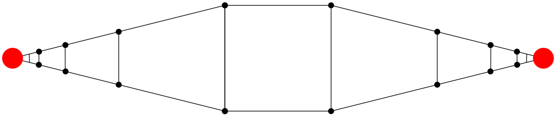

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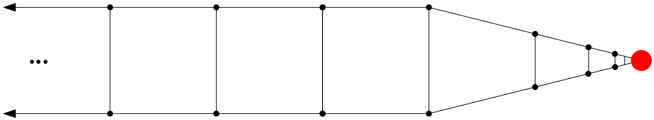

Axioms for finite matroids (Minty)

$\mathcal{C}$ and $\mathcal{D}$ are respectively the sets of circuits and of cocircuits of a matroid on a finite set E if and only if

- (C1) $\emptyset \notin \mathcal{C}$
- (C2) No proper subset of an element of $\mathcal{C}$ is in $\mathcal{C}$
- (C1*) $\emptyset \notin \mathcal{D}$
- (C2*) No proper subset of an element of $\mathcal{D}$ is in $\mathcal{D}$
- (O1) For $C \in \mathcal{C}$ and $D \in \mathcal{D}$ the set $C \cap D$ never has just 1 element.
- (O2) For any partition of E as $P \dot{\cup} Q \dot{\cup} \{e\}$, either there is $C \in \mathcal{C}$ with $e \in C \subseteq P + e$ or else there is $D \in \mathcal{D}$ with $e \in D \subseteq Q + e$.

Axioms for countable tame matroids

$\mathcal{C}$ and $\mathcal{D}$ are respectively the sets of circuits and of cocircuits of a tame matroid on a countable set E if and only if

(C1) $\emptyset \notin \mathcal{C}$

(C2) No proper subset of an element of $\mathcal{C}$ is in $\mathcal{C}$

(C1*) $\emptyset \notin \mathcal{D}$

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(O1) For $C \in \mathcal{C}$ and $D \in \mathcal{D}$ the set $C \cap D$ never has just 1 element.

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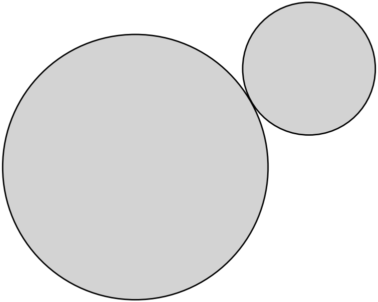
(T) $C \cap D$ is finite for any $C \in \mathcal{C}$ and $D \in \mathcal{D}$.

Gluing matroids together



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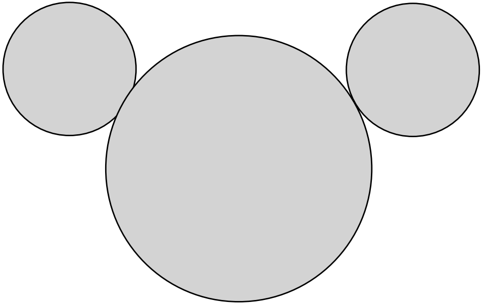

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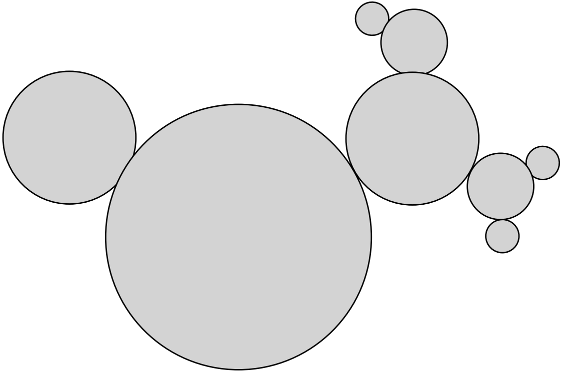

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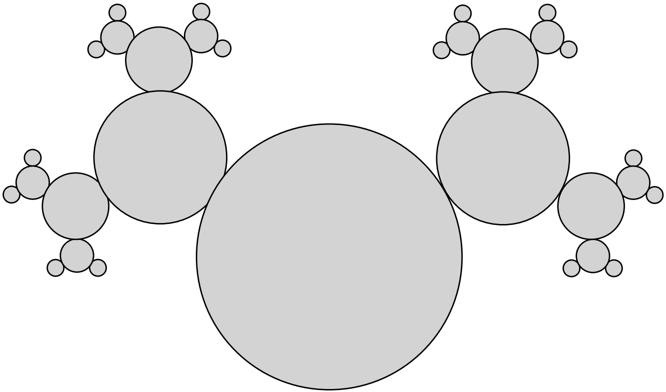

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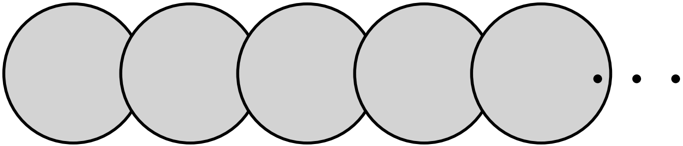

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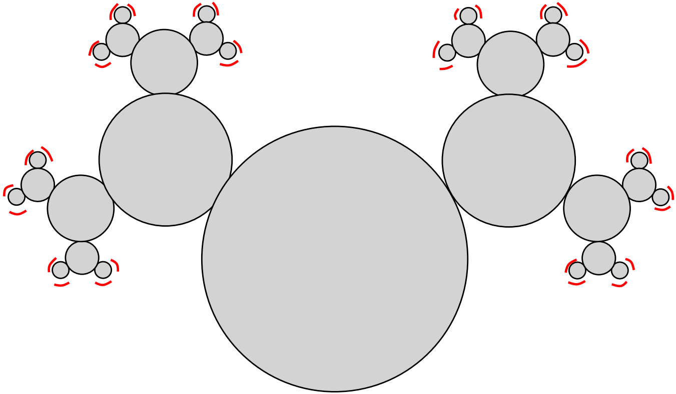

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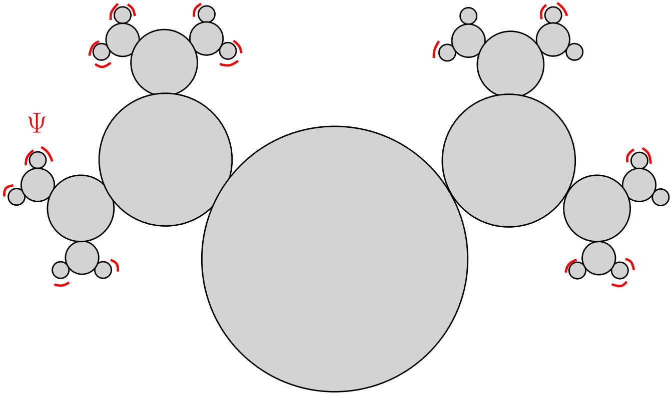

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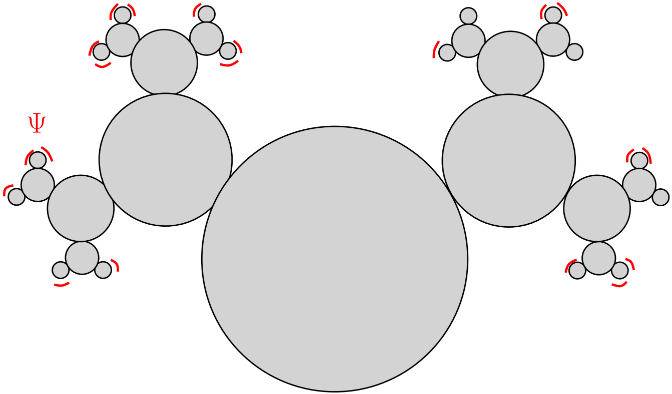
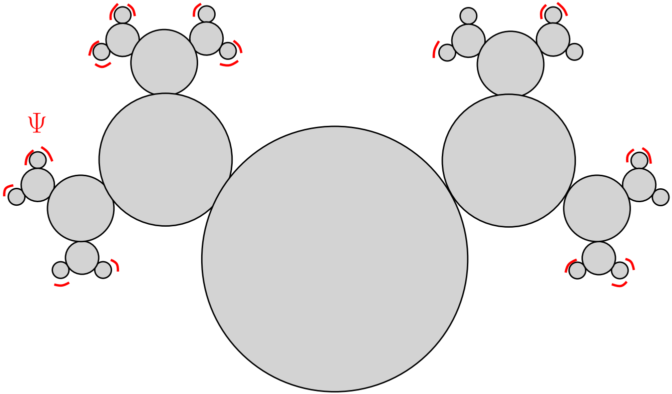


Theorem

If the set Ψ is Borel then $M_\Psi(\mathcal{T})$ is a matroid.

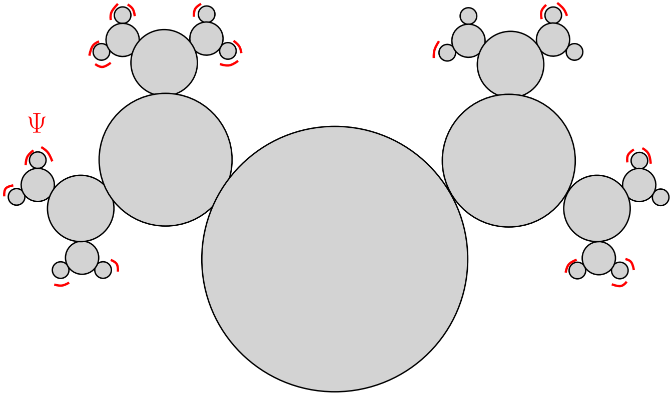
Gluing matroids together



Theorem (Martin)

If a set Φ is Borel then it is determined.

Gluing matroids together



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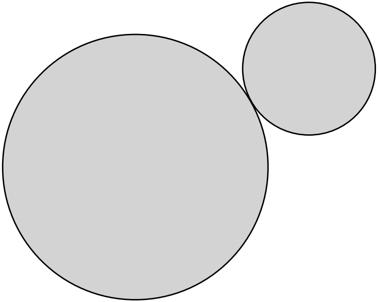

G-matroids are Ψ-matroids.

Definition

Let G be a locally finite graph. A *G-matroid* is a tame matroid all of whose circuits are topological circles of G and all of whose cocircuits are bonds of G.

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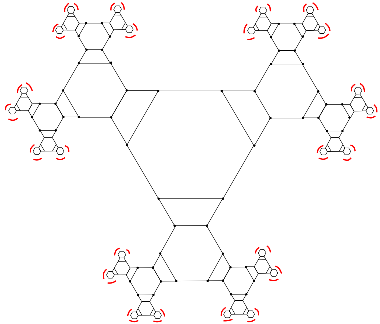

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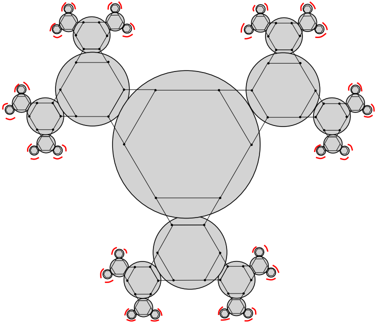

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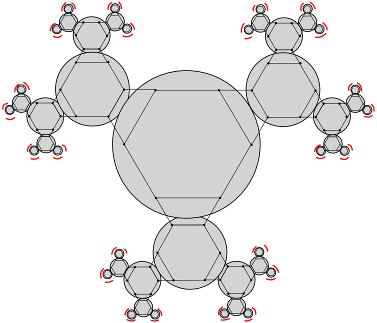

Theorem

There is a decomposition of G into a tree $\mathcal{T}$ of finite graphs such that every G-matroid is of the form $M_\Psi(\mathcal{T})$.

Rebuilding matroids out of their 3-connected parts

Theorem (Tutte)

Any finite matroid can be canonically represented as a 2-sum of a tree of matroids, each of which is either 3-connected, a single circuit or a single cocircuit.

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Any matroid can be canonically decomposed over its 2-separations into a tree of matroids, each of which is either 3-connected, a single circuit or a single cocircuit.

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Theorem (Aigner-Horev, Diestel and Postle)

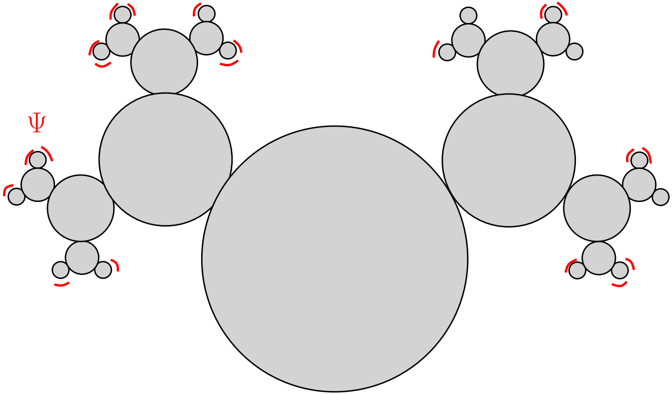
Any matroid can be canonically decomposed over its 2-separations into a tree of matroids, each of which is either 3-connected, a single circuit or a single cocircuit.

Theorem

Any tame matroid can be canonically represented as $M_\Psi(\mathcal{T})$ for some tree $\mathcal{T}$ of tame matroids each of which is either 3-connected, a single circuit or a single cocircuit.

THANKS FOR LISTENING!

Why determinacy is relevant



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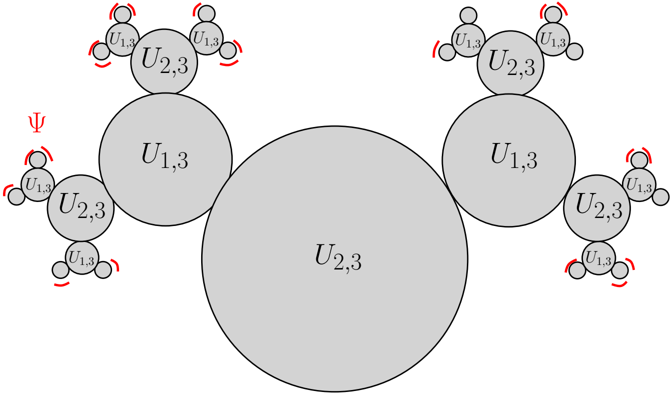

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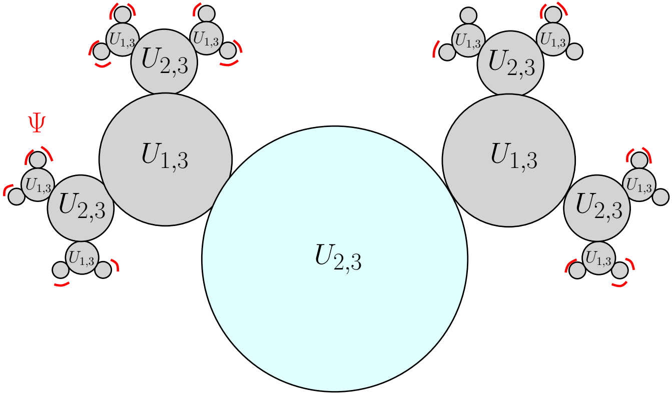

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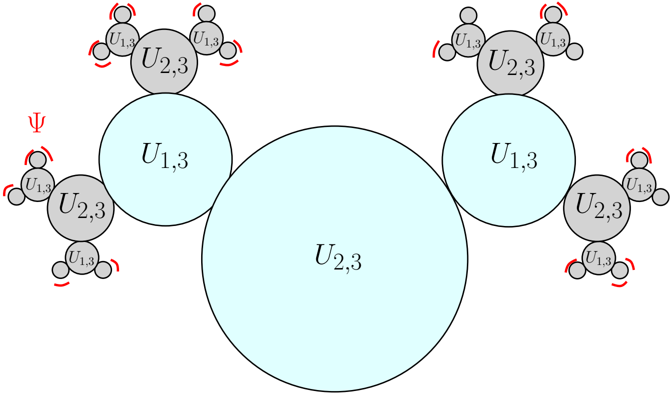

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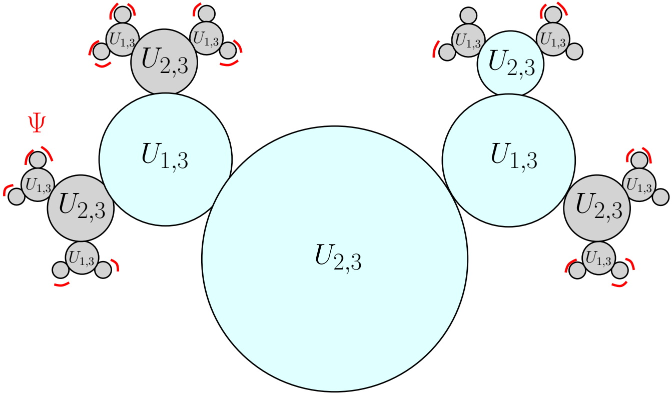

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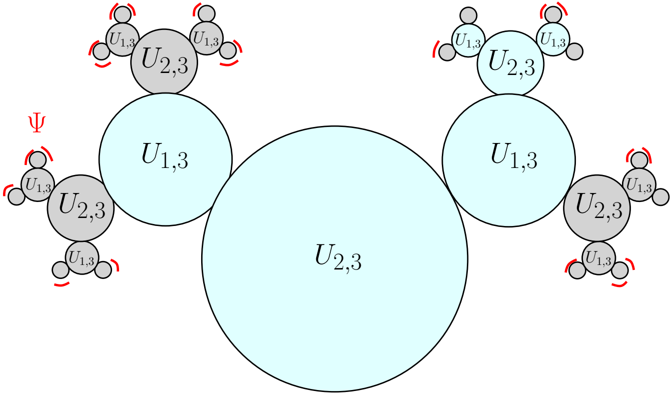

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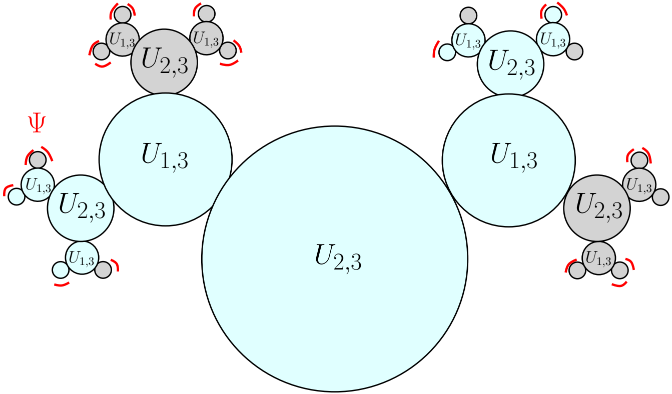

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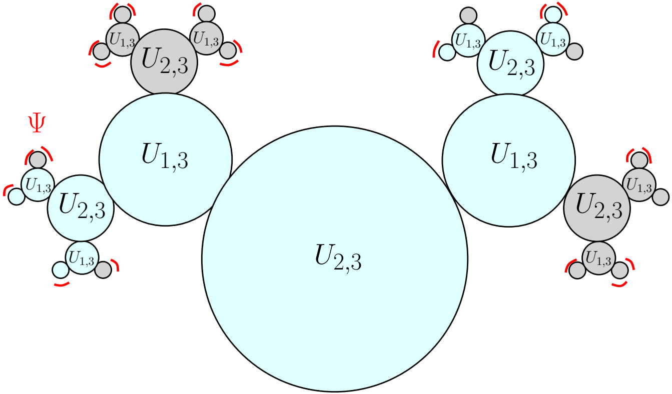

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If the set Ψ *is Borel then the game* $G(\Psi)$ *is determined.*

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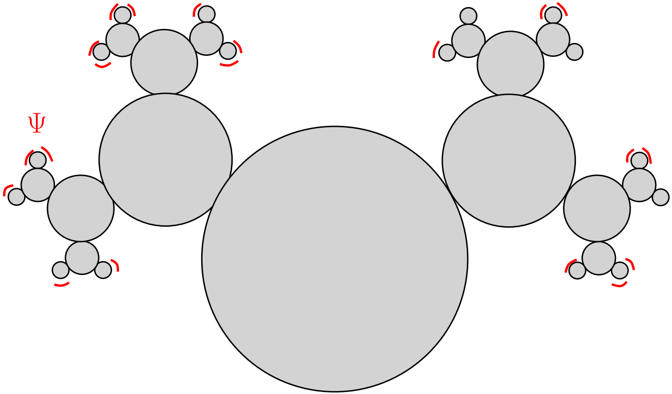

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