Infinite trees of matroids

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\textsuperscript{1}Joint work with Johannes Carmesin
Examples of infinite matroids

Two matroids from a (locally finite) graph $G$, given in terms of their circuits:

Finite cycle matroid $M_{FC}(G)$: finite cycles
Topological cycle matroid $M_{C}(G)$: topological circles in $|G|$
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Axioms for finite matroids (Minty)

$\mathcal{C}$ and $\mathcal{D}$ are respectively the sets of circuits and of cocircuits of a matroid on a finite set $E$ if and only if

(C1) $\emptyset \notin \mathcal{C}$

(C2) No proper subset of an element of $\mathcal{C}$ is in $\mathcal{C}$

(C1*) $\emptyset \notin \mathcal{D}$

(C2*) No proper subset of an element of $\mathcal{D}$ is in $\mathcal{D}$

(O1) For $C \in \mathcal{C}$ and $D \in \mathcal{D}$ the set $C \cap D$ never has just 1 element.

(O2) For any partition of $E$ as $P \cup Q \cup \{e\}$, either there is $C \in \mathcal{C}$ with $e \in C \subseteq P + e$ or else there is $D \in \mathcal{D}$ with $e \in D \subseteq Q + e$. 

$\Box$
Axioms for countable tame matroids

$C$ and $D$ are respectively the sets of circuits and of cocircuits of a tame matroid on a countable set $E$ if and only if

(C1) $\emptyset \not\in C$

(C2) No proper subset of an element of $C$ is in $C$

(C1*) $\emptyset \not\in D$

(C2*) No proper subset of an element of $D$ is in $D$

(O1) For $C \in C$ and $D \in D$ the set $C \cap D$ never has just 1 element.

(O2) For any partition of $E$ as $P \dot\cup Q \dot\cup \{e\}$, either there is $C \in C$ with $e \in C \subseteq P + e$ or else there is $D \in D$ with $e \in D \subseteq Q + e$.

(T) $C \cap D$ is finite for any $C \in C$ and $D \in D$. 

Theorem (Martin)
If a set $\Phi$ is Borel then it is determined.
Gluing matroids together

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Theorem

If the set $\Psi$ is Borel then $M_\Psi(T)$ is a matroid.
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Gluing matroids together

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If the set $\Psi$ is Borel then $M_\Psi(\mathcal{T})$ is a matroid.
Gluing matroids together
**Definition**

Let $G$ be a locally finite graph. A $G$-matroid is a tame matroid all of whose circuits are topological circles of $G$ and all of whose cocircuits are bonds of $G$. 

$G$-matroids are $\Psi$-matroids.
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**Definition**
Let $G$ be a locally finite graph. A $G$-matroid is a tame matroid all of whose circuits are topological circles of $G$ and all of whose cocircuits are bonds of $G$.

**Theorem**
There is a decomposition of $G$ into a tree $T$ of finite graphs such that every $G$-matroid is of the form $M_\Psi(T)$. 
Rebuilding matroids out of their 3-connected parts

Theorem (Tutte)
Any finite matroid can be canonically represented as a 2-sum of a tree of matroids, each of which is either 3-connected, a single circuit or a single cocircuit.
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Theorem (Aigner-Horev, Diestel and Postle)

Any matroid can be canonically decomposed over its 2-separations into a tree of matroids, each of which is either 3-connected, a single circuit or a single cocircuit.
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Any matroid can be canonically decomposed over its 2-separations into a tree of matroids, each of which is either 3-connected, a single circuit or a single cocircuit.

Theorem
Any tame matroid can be canonically represented as $M_\Psi(T)$ for some tree $T$ of tame matroids each of which is either 3-connected, a single circuit or a single cocircuit.
THANKS FOR LISTENING!
Why determinacy is relevant

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If the set $\Psi$ is Borel then the game $G(\Psi)$ is determined.
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Theorem

If the set $\Psi$ is Borel then $M_{\Psi}(T)$ is a matroid.