Unbreakable Matroids

Preliminary Report

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2014 International Workshop on Structure in Graphs and Matroids

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Definition

A matroid M is called *unbreakable*
if, for all flats F of M, the matroid M/F is connected.

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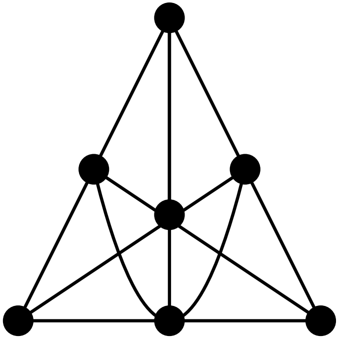

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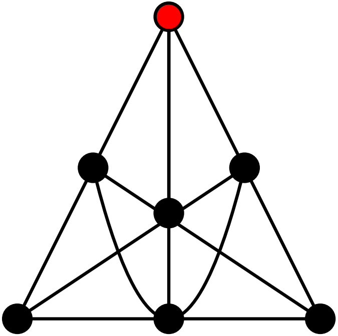

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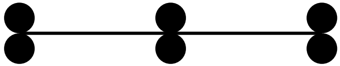

Some Useful Terms.

Definition

The *local connectivity* of two subsets X and Y of a matroid M, denoted $\sqcap_M(X, Y)$, is defined
$\sqcap_M(X, Y) = r_M(X) + r_M(Y) - r_M(X \cup Y)$.

Definition

Subsets X and Y of a matroid M are *skew* if $\sqcap_M(X, Y) = 0$.

Definition

A matroid N is a *parallel minor* of M if N can be obtained from M by a sequence of deletions of elements in parallel and contractions.

The Dual Definition.

Proposition

M is unbreakable if, and only if, M^* has no skew circuits.

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Proof.

$$M/F \text{ disconnected} \iff M^* \backslash F \text{ disconnected}$$

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\begin{aligned}
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&\iff M^*|(E(M) - F) \text{ disconnected}
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Equivalent Characterizations

Proposition

M is unbreakable if, and only if, M/f is unbreakable for all rank-1 flats f.

Proposition

M is unbreakable if, and only if, M has no $U_{2,2}$ parallel minor.

Some Examples.

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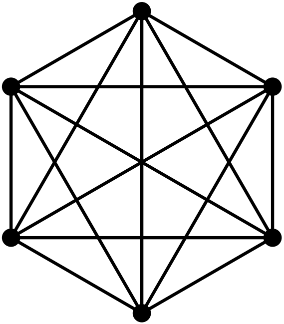

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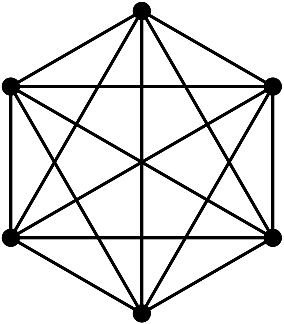

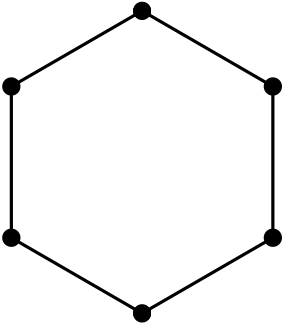

Theorem Classifying Unbreakable Graphic Matroids.

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A graphic matroid $M(G)$ is unbreakable if, and only if, $si(G) \cong C_n$ or K_n.

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- Find non-planar graphic matroids without skew circuits.

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- Use the dual definition.
- Find non-planar graphic matroids without skew circuits.
- Find graphs such that every pair of cycles share at least two vertices.

Unbreakable Cographic Matroids.

Theorem (Dirac)

Every 3-connected graph with no two vertex disjoint cycles is one of the following graphs:
W_k $(k \geq 4)$, K_5, K_5^-, $K_{3,p}$, $K'_{3,p}$, $K''_{3,p}$, $K'''_{3,p}$, $(p \geq 3)$.

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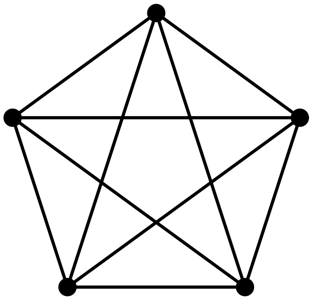

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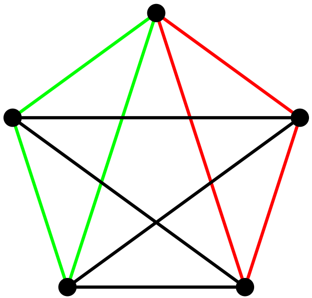

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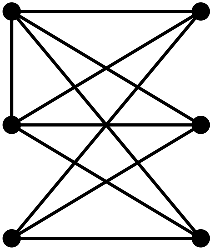

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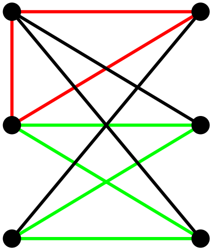

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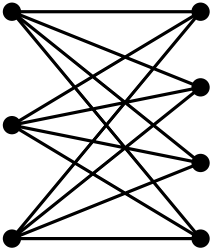

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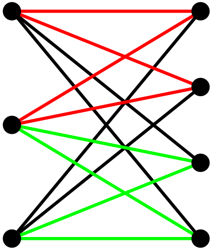

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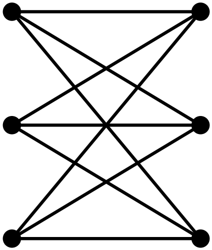

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Theorem

A cographic matroid M is unbreakable if, and only if, $si(M) \cong M(C_n), M(K_2), M(K_4)$, *or* $M^*(K_{3,3})$.

Seymour's Decomposition Theorem

Theorem (Seymour)

A regular matroid M can be constructed using 1-, 2-, and 3-sums of matroids which are either graphic, cographic, or R_{10}, and each of which is isomorphic to a minor of M.

R_{10} is Unbreakable.

$$R_{10} = M\left(\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}\right)$$

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- R_{10} has rank 5.
- R_{10} is self-dual.
- The smallest circuit in R_{10} is size 4.

Preliminary Lemmas.

Lemma

The matroid $(M_1, p) \bigoplus_2 (M_2, p)$ *is unbreakable if, and only if, p is a free element in both.*

Definition

An element e of M is called *free* if the only circuits of M containing e are spanning.

Proposition

Let M_1 *and* M_2 *be binary matroids with* $E(M_1) \cap E(M_2) = T$*, where* $M_1|T = M_2|T$ *is a triangle. If* $e \in E(M_1) - cl_1(T)$*, then* $(M_1 \bigoplus_3 M_2)/e = (M_1/e) \bigoplus_3 M_2$*.*

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- What about 3-sum?

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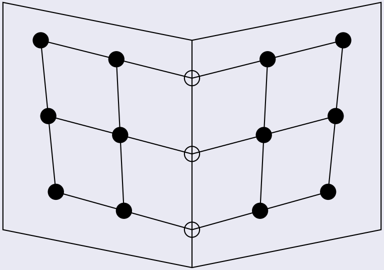

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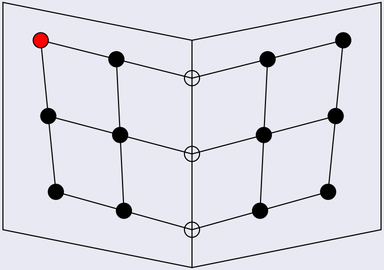

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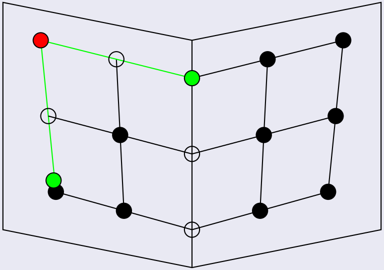

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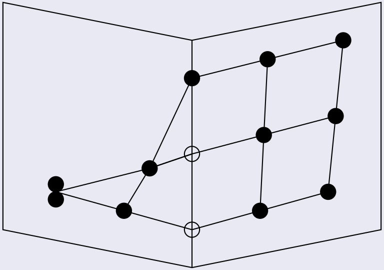

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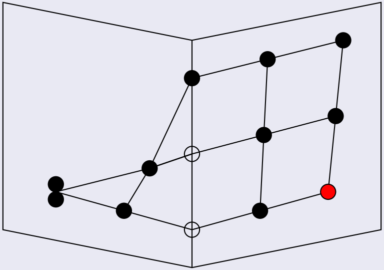

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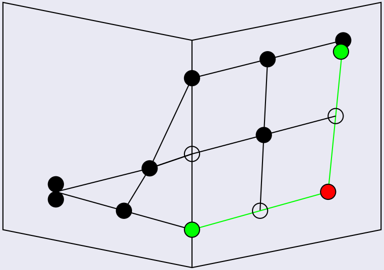

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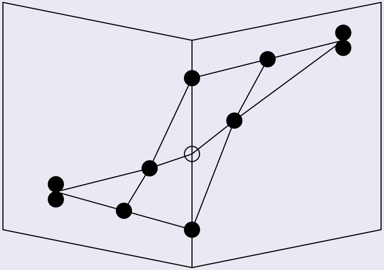

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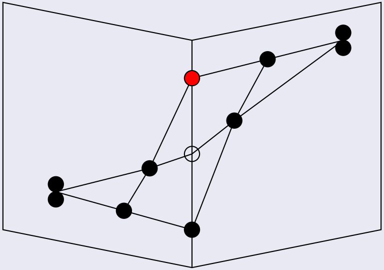

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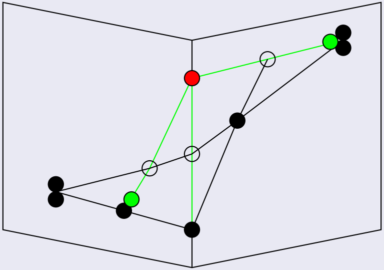

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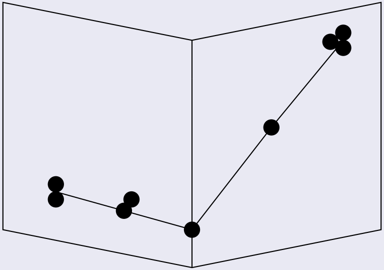

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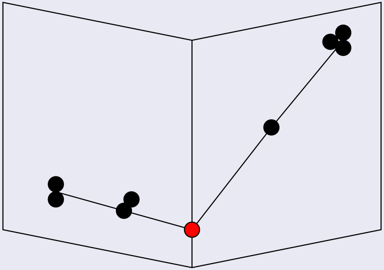

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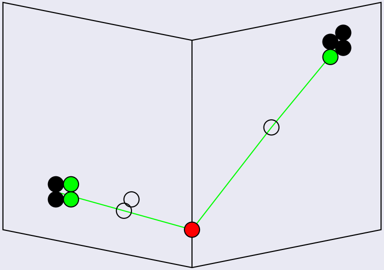

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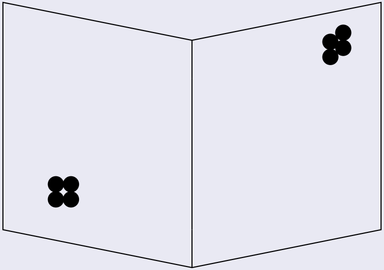

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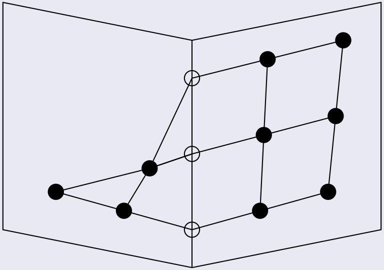

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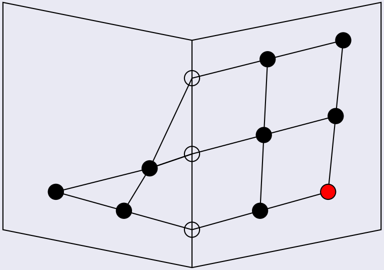

Theorem Classifying Unbreakable Regular Matroids.

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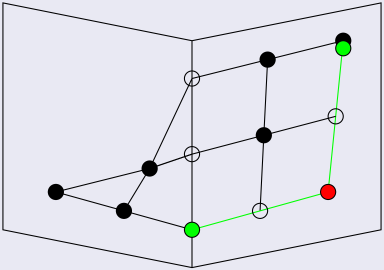

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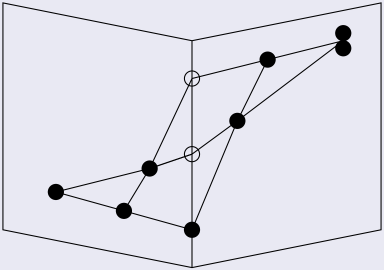

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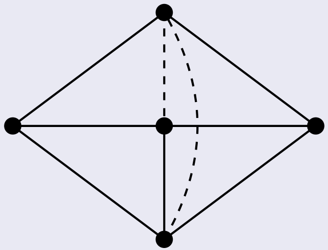

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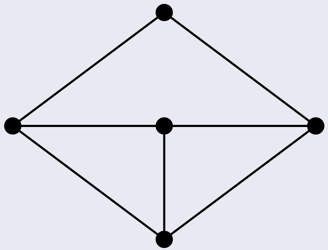

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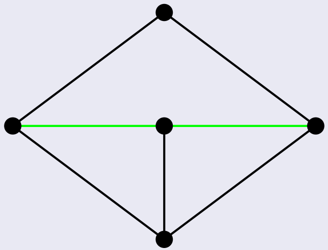

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A regular matroid M is unbreakable if, and only if, $si(M) \cong M(C_n), M(K_n), M^*(K_{3,3})$, or R_{10}.

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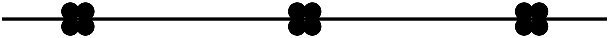

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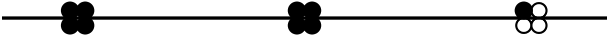

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- Similarly, $AG(r-1,2)$ is not minimal.

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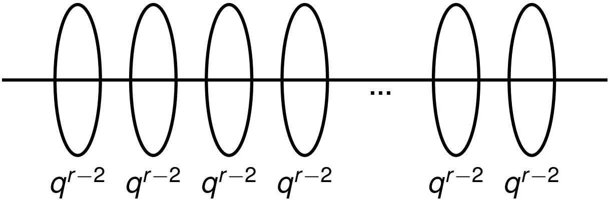

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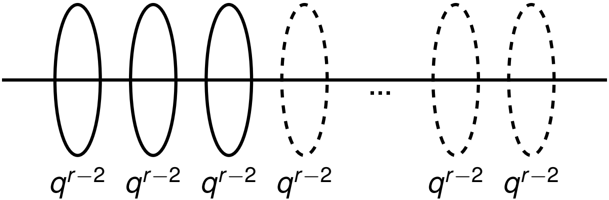

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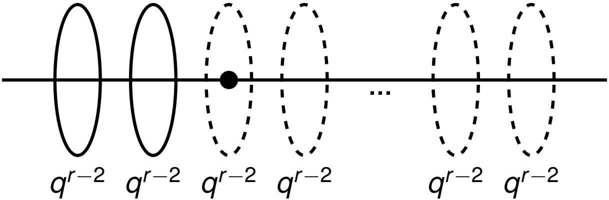

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- Delete $q^{r-2}(q-2) + (q^{r-2} - 1)$ elements, maintain unbreakability.
- The largest rank-r, $GF(q)$ representable matroid that is not unbreakable has $q^{r-2} + 1 + \sum_{i=0}^{r-2} q^i$ elements.

Thank you!

Round vs. Unbreakable.

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- $(X, Y) = E(M)$, $X' \subset X$ *is a flat, and* $Y' \subseteq Y$*, then* $\sqcap(X', Y') < \sqcap(X, Y)$.

When Can "Round’ be Replaced by "Unbreakable"?